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Aerodynamic space tether system as a system with distributed parameters

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Abstract. The motion of the aerodynamic tether system includes two stages – the deployment and the motion of the deployed system. The initial condition for the deployment model is that of a spacecraft with a rigidly connected second body in a circular orbit about the planet. The deployment process is based on the aerodynamic forces acting on bodies. After the separation of the rigid connection, further motions of the bodies are then controlled by the tether release mechanism which unreels the tether and controls the rate at which the tether is unreeled. The motion of the system is described by the multi-point model, splitting the tether into a number of parts and using the series of separations of material points from the spacecraft, thus forming a mathematical model of the system with distributed parameters. The aerodynamic forces acting on the tether are taken into consideration. This model makes it possible to calculate numerically elastic deformations and curvature of the tether and provides more accurate modelling if compared with the two-point model.

1. Introduction

The aerodynamic space tether system is a system of two rigid bodies connected by a long thin stretchable tether where the aerodynamic forces has a major influence on the motion of the system. The orientation, or attitude, is the part of the description of the placement that the rigid body occupies in space. The other component of placement is its linear position. The attitude is defined by the angles between the longitudinal axis of the spacecraft and some reference line or plane [1] such as the vector of velocity of the air flow. Therefore, stabilization means the process of maintaining attitude during motion or tending to restore the original attitude after receiving a displacement or perturbation. For a spacecraft, attitude control is the process of achieving and maintaining a desired orientation in space by a corrective action.

Aerodynamic space tether systems can be used for a wide number of applications and purposes. The most obvious application is to provide the appropriate orientation of a spacecraft during descent. This method does not require energy expense, works based on aerodynamic forces, and requires only the presence of the atmosphere. Therefore, the aerodynamic stabilization, using the tether system, can be used not only during descent to Earth, but to stabilize the motion before landing onto the surface of other planets.

The aerodynamic tether system is applicable for returning payloads from outer space to the surface of the planet, especially using small landing modules from the space station. The payload in small modules can include, but is not limited to, materials manufactured in the absence of gravity, devices for service and maintenance, or examples of space objects. In this case, the tether system is helpful in lowering the orbit of the landing module before descent and enabling stable motion during the second stage of descent. Tethered aerodynamic stabilization can be used to target the detachable stages of rocket launchers after their separation from the launcher with greater accuracy towards the desired landing site area. The problem of reducing the search area over which detachable parts of the launchers might land has a great economic and ecological significance, because without aerodynamic stabilization these areas can cover up to thousands of square kilometres.

The most comprehensive investigations of tether systems were associated with National Aeronautics and Space Administration (NASA) [2], where the models of motion and deployment are described, as well as results of experiments on tether systems. Estimates of heating and deformation of the tether under the influence of gravity forces are included.

Tether systems are applicable to investigations of the higher layers of the atmosphere, where the flight of traditional aircraft is not possible [3, 4], to stabilize the motion of spacecraft [5], for removing space debris [6, 7] and for deep space investigations [8]. A tether system with an atmospheric probe is a perspective technology for gathering important information about higher layers of the atmosphere of Earth, at altitudes about 100 to 150 km. The density of the atmosphere at these heights is too low for normal aircraft operation, while on the other hand, traditional spacecraft would meet comparatively high aerodynamic drag. This problem can be solved by using an atmosphere probe, connected by tether to a spacecraft operating at an altitude of about 200 km [9]. It is necessary to mention that direct current can lead to motion instability in the tether system, and that, by changing the amperage in the tether, it is possible to stabilize the motion of the tether system on the electrodynamic basis [10, 11]. A tether system can provide deceleration of a spacecraft before landing [12], but it should be noted that in this case the tether tension force is significantly higher than for upper atmospheric use [13].

2. Mathematical models of the tether system

The tether system is a complex non-linear dynamic system where parameters are distributed. In general, the tether is a flexible mechanical connection which has a distributed mass along its length. Within the tether, there are complex longitudinal, bending and torsion oscillations. The investigations of tether systems focus on two major processes. Firstly, it is the motion of the deployed tether system [14], and secondly, the process of deployment of a tether system [15].

Five types of tether mathematical models were considered here:

1. The *Rigid Rod Model* is a simple model of the tether that ignores its extensibility and flexibility. In other words, the tether has in infinite rigidity. This model was used for an evaluation of the possibility to use tether system for atmosphere investigations [16] and to model the motion of a tether system with a relatively short tether in the atmosphere [13].
2. The *Elastic Rod Model*. Like in the previous model, the tether is a straight rod, but it can be stretched axially. This model ignores the flexibility of the tether, but the extensibility is taken into consideration [17-18].
3. The *Billiard Model* takes into consideration both the axial stretch and bending flexibility of the tether. The tether in this model is a weightless flexible spring. The model allows the bending oscillations to be described and enables limitations on transversal displacements of the tether to be made [19]. This model is inappropriate to describe the dynamics of parts of the tether but makes it possible to research the motion of the tether system in gravity field of the planet.
4. The model of *Rigid Rods with Swivel Joints* was used in [20] to find the parameters for the optimal deployment of the tether. This model has zero torsion stiffness, does not take into consideration the longitudinal oscillations and encounters difficulties in describing the sag of the tether.
5. The *Multi-point Model* describes the tether as a series of material points, also called nodes. The nodes have masses but are connected to neighbouring nodes by weightless springs. The number of points is defined during the mathematical modelling. The more intermediate points included into the mathematical model, the more accurate the model; however, increasing the number of nodes leads to increase in number of degrees of freedom thus greatly increasing the number of

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equations and time required for calculations. Therefore, this model commonly used to validate other models [21-23].

Mathematical models of the tether system can be obtained using different but equivalents methods, including the Lagrange Equations, Newton's Laws [24] and Kane's method, originally called Lagrange's form of the D'Alembert principle [25]. The complexity of mathematical models of the tether systems leads to the time-consuming calculations. These time expenses can be unreasonably high during optimization tasks, and can be reduced by using various numerical schemes, such as the artificial increase of periods of fast oscillations in the mathematical model [26].

On modelling a deployed tether system motion in the atmosphere, it can be shown that by changing the parameters of the stabilizer it is possible to obtain a stable tether system motion [14]. In general, the tension of the tether depends most significantly on the geometrical dimensions of the bodies, while length of the tether has only a slight influence on this force. The increase of the tether length during the descent always leads to increased stability of the tether system motion. This leads to a reduction in the amplitudes of the angle of attack oscillations for the bodies and the tether. Therefore, the deployment of a tether during descent can increase the stability of the system.

3. Modelling the motion of the aerodynamic tether system

3.1. Definitions

Stable motion is defined as the pre-defined orientation of the tether system such that the angles between the longitudinal axis of bodies and tether do not exceed 90°. The angles of attack of the bodies and the tether should also be less than 90°. The initial condition for the mathematical model is that the tether system is in a circular orbit at an altitude of about 250 km prior to the separation of the *Spacecraft* (first body) and the second body, called here the *Stabilizer*.

The parameters for the body moving through the air of the upper atmosphere include the mass, the reference area and the drag coefficient. It is possible to define the ballistic coefficient of the rigid body as the combinations of these parameters, $\sigma = C_{xv}S/m$, where C_{xv} is the drag coefficient, S is the reference area, and m is mass. Thus, through the design of the tether system, the stabilizer has a significantly higher ballistic coefficient in comparison with the spacecraft. The mathematical model takes into consideration the extensibility of the tether so that the influence of deployment method on the motion of the system can be estimated. The mass and geometrical asymmetries of the spacecraft and the stabilizer, which are modelled as rigid bodies, are also taken into consideration. The aerodynamic deceleration during the descent from planetary orbit is included within the mathematical model.

The separation of bodies starts with a comparatively low relative velocity $\vec{V}_r$. After the separation of the rigid connection between the spacecraft and stabilizer, their further motion is then controlled by the tether release mechanism. The tether release mechanism unreels the tether but is not able to pull the tether back in, therefore the equations of motion of the system include a monotonic deceleration term representing the tether release mechanism. Specific features of regulation system such as discrete work, design tolerances, *etc.*, are not taken into consideration. The tether itself works in tension only. The deployment of the stabilizer body is controlled through the deceleration of the tether. The deceleration control device is situated on the spacecraft.

3.2. Geometry and Equations of Motion

The mathematical model describes the process of deployment as well as further motion of the tethered system and includes a perturbation analysis. The aerodynamic resistance of the tether and its mass characteristics are both taken into consideration in the calculation. The mathematical model of the motion during deployment uses the geocentric coordinate system $Oxyz$, which is bound to the plane of the orbit of the centre of mass of the tether system. The plane of the orbit is defined at the moment of the separation of the stabilizer from the spacecraft. The Ox axis is directed to the ascending node of the orbit, the Oz axis is parallel to the vector of total angular momentum of the centre of mass of the system. The spacecraft and stabilizer are rigid bodies of finite length, which are connected by the tether (figure 1). The coordinate systems associated with the bodies $Cx_iy_iz_i$ are used to define the moments of inertia for each of the bodies (figure 2).

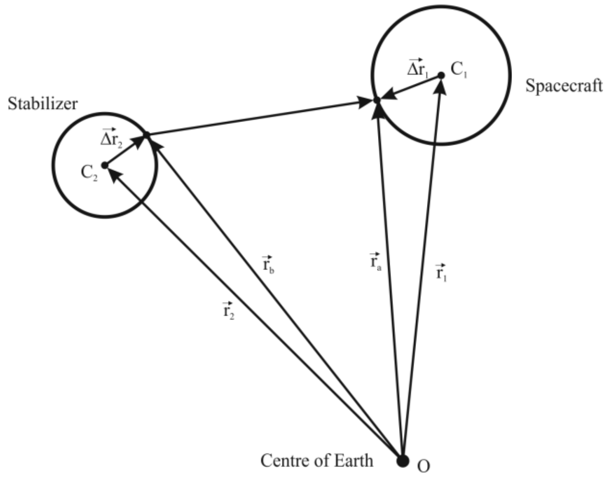

Figure 1. Tether system.

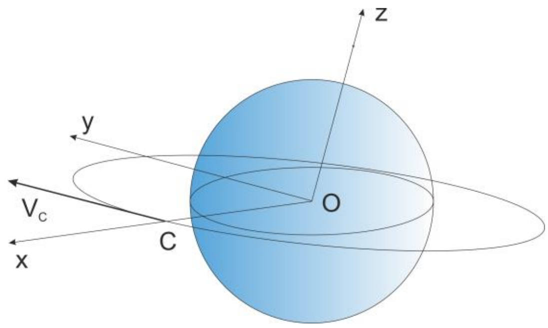

Figure 2. Coordinates system.

The stability is considered lost when the angles ψ_i between longitudinal axis C_ix_i and vector $\vec{r}_{ab}$ or when the angles of attack α_i become higher than $\pi/2$. The ideal orientation of the tether system means that the longitudinal axes C_ix_i and the tether are co-linear with the vector of the velocity of the centre of mass of the tether system. Thus, the angle between the vector of the tether and the vector of velocity of the centre of mass defines the orientation of the spacecraft.

The equations of motion of this system include the equations for the motion of the centres of mass

$$m_i \frac{d^2\vec{r}_i}{dt^2} = \vec{g}_e(r_i) + \vec{T}_i + \vec{R}_i \tag{1}$$

and the equations of rotational motion of the bodies

$$\begin{aligned} I_{xi}\frac{d\omega_{xi}}{dt} + \omega_{yi}\omega_{zi}\left(I_{zi} - I_{yi}\right) = l_{xi}, \\ I_{yi}\frac{d\omega_{yi}}{dt} + \omega_{xi}\omega_{zi}\left(I_{xi} - I_{zi}\right) = l_{yi}, \\ I_{zi}\frac{d\omega_{zi}}{dt} + \omega_{xi}\omega_{yi}\left(I_{yi} - I_{xi}\right) = l_{zi}, \end{aligned} \tag{2}$$

where indexes $i = 1$ and $i = 2$ refer to the spacecraft and the stabilizer respectively, m_i are masses and r_i are radius-vectors of centres of masses for both bodies, $\vec{g}_e(r_i)$ and $\vec{R}_i$ are gravitational and aerodynamic forces, t is the elapsed time during tether deployment starting from the instant of separation, $\vec{T}_i$ is the tension force of the tether, $\vec{T}_1 = -\vec{T}_2$, I_{xi}, I_{yi}, and I_{zi} are the moments of inertia in associated with the coordinate systems $Cx_iy_iz_i$ of the bodies, ω_{xi}, ω_{yi}, ω_{zi} are components of angular velocities of the bodies, and l_{xi}, l_{yi}, l_{zi} are components of the net total angular momentum that act on each of the bodies.

3.3. Forces applied to the Tether System

In the modelling of the motion of the system, the torques from aerodynamic forces and torques arising from the tension forces are taken into consideration. The torques from gravity forces have not been included because they can be shown to be negligibly small. The tension force is defined by Hooke's law for one-side mechanical connection.

$$\vec{T}_2 = T\frac{\vec{r}_{ab}}{r_{ab}},$$

where $\vec{r}_{ab} = \vec{r}_a - \vec{r}_b$ (as shown in figure 1), $\vec{r}_a$ and $\vec{r}_b$ are radius vectors for the tether attachment points. The value of T is equal to zero when r_{ab} is less than the length L_{ND} of non-deformed tether, and $T = k\,(\mathrm{r_{ab}} - L_{\mathrm{ND}})/L_{\mathrm{ND}}$ when $\mathrm{r_{ab}} \geq L_{\mathrm{ND}}$, where k is the spring constant of the tether.

If the tether is not strained, both bodies move freely under the action of aerodynamic forces. For modelling purposes, it is presumed that the motion takes place in low density gas and the hypothesis of diffuse reflection of gas molecules is applicable. Based on this hypothesis, vectors of the aerodynamic forces are co-linear with the vectors of velocities of the bodies defined relative to the atmosphere

$$\vec{R}_i = -\frac{C_i S_i \rho V_i \vec{V}_i}{2}, \quad (3)$$

where C_i are drag coefficients, $\rho = \rho(h)$ is the density of the atmosphere and is function of the altitude h, $S_i = S_i(\alpha_i)$ is the square of projection of the body onto the plane which is perpendicular to the velocity vector $\vec{V}_i$, α_i is the angle of attack during the spatial motion, $i = 1, 2$. If the bodies are spherical, S_i is constant.

The velocities $\vec{V}_i$, calculated relative to the atmosphere, can be found using the equation

$$\vec{V}_i = \frac{d\vec{r}_i}{dt} - \vec{\omega}_{\mathrm{Earth}} \times \vec{r}_i,$$

where $\vec{\omega}_{\mathrm{Earth}}$ is the angular velocity of rotation of the Earth.

The gravity forces are defined by Newton's law

$$\vec{g}_e(r_i) = -G\frac{M_E m_i}{|r_i|^2}\vec{r}_i,$$

where G is the gravity constant, M_E is the mass of the Earth.

The dynamics of the tether regulating mechanism are taken into account in the modelling of the deployment process, as follows

$$m_u \frac{d^2 r_3}{dt^2} = T - F_{\mathrm{control}}(t), \quad (4)$$

where r_3 is the length of the tether, the constant coefficient m_u describes the inertness of the mechanism, $F_{\mathrm{control}}(t)$ is the control force. Then the release of the tether is unguided, the control decelerating force is constant, positive and close to zero. For the guided deployment, the tether release mechanism works on deceleration only, $dr_3/dt \geq 0$. The control force is also greater than zero, $F_{\mathrm{control}}(t) \geq F_{\min} > 0$, where $F_{\min}$ is the minimum deceleration force that the tether release mechanism can provide.

3.4. The Kinematic Equations

The equations of motion of the system should be supplemented by kinematic equations. Here, the Euler angles are used for both bodies in the coordinate systems associated with vectors $\vec{V}_i$ and $\vec{r}_i$. The unit vectors for these systems of coordinates are

$$\vec{e}_{1i} = \vec{V}_i / V_i \qquad \vec{e}_{3i} = \vec{V}_i \times \vec{r}_i / \left|\vec{V}_i \times \vec{r}_i\right| \qquad \vec{e}_{2i} = \vec{e}_{3i} \times \vec{e}_{1i}. \tag{5}$$

The kinematics equations for Euler angles are

$$\omega_{xi} = \dot{\varphi}_i + \dot{\gamma}_i \cos\alpha_i + \Delta\omega_{xi},$$
$$\omega_{yi} = \dot{\alpha}_i \sin\varphi_i - \dot{\gamma}_i \sin\alpha_i \cos\varphi_i + \Delta\omega_{yi},$$
$$\omega_{zi} = \dot{\alpha}_i \cos\varphi_i + \dot{\gamma}_i \sin\alpha_i \sin\varphi_i + \Delta\omega_{zi},$$

where, $\Delta\omega_{xi}$, $\Delta\omega_{yi}$, and $\Delta\omega_{zi}$ are the corrections to the angular velocities due to the rotation of coordinate systems (5),

$$\begin{pmatrix} \Delta\omega_{xi} \\ \Delta\omega_{yi} \\ \Delta\omega_{zi} \end{pmatrix} = L_i^T \begin{pmatrix} 0 \\ 0 \\ \omega_{vi} \end{pmatrix} \quad \text{where} \quad \begin{gathered} \omega_{vi} = V_i / r_i \, sin\vartheta_i \\ \vartheta_i = \text{acos}\left(\frac{\vec{r}_i \cdot \vec{V}_i}{r_i V_i}\right), \\ L_i = L_{\varphi i} L_{\alpha i} L_{\gamma i}, \end{gathered}$$

$$L_{\varphi i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_i & \sin\varphi_i \\ 0 & -\sin\varphi_i & \cos\varphi_i \end{pmatrix}, \quad L_{\alpha i} = \begin{pmatrix} \cos\alpha_i & \sin\alpha_i & 0 \\ -\sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad L_{\gamma i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_i & \sin\gamma_i \\ 0 & -\sin\gamma_i & \cos\gamma_i \end{pmatrix}.$$

After the separation of the stabilizer from the spacecraft, it is necessary to recalculate their velocities using the conservation of linear momentum law

$$\vec{V}_1 = \vec{V}_C - \frac{m_2}{m_1 + m_2} \vec{V}_2, \qquad \vec{V}_2 = \vec{V}_C + \vec{V}_r, \tag{6}$$

where $\vec{V}_C$ is the velocity of the centre of mass of the tether system, $\vec{V}_1$ and $\vec{V}_2$ are velocities of the spacecraft and the stabilizer after the separation in geocentric coordinate system.

3.5. Deployments Models

The implementations of space tether systems require a method for the control of the deployment process. Tether systems with non-electrical tethers use rocket engines for centre of mass motion correction, and special devices to control the unreeling the tether [27]. The control of the unreeling can be with or without feedback. The simplest non-feedback method of the deployment is release of the tether with a minimal decelerating force until the tether is completely unreeled. The deployment can be realized with a decreasing rate of change of tether length basing on the continuous deceleration or basing on the Length rate of change control. A dynamic deployment method with constant decelerating force can be improved by mitigating the limitations of the final tether length and by using the program dynamic method, which includes regions of both acceleration and deceleration.

A dynamic method with constant decelerating force and a programmable change of control force can be improved by making use of the feedback principle. In this instance, the following form of regulating force can be applied

$$F_{\text{control}} = F_n(t) + p_L\left[L - L_p(t)\right] + p_V\left[\dot{L} - \dot{L}_p(t)\right] \tag{7}$$

where $L_p(t)$ and $\dot{L}_p(t)$ are pre-defined program dependencies of tether length and the rate of change of length with respect to time; p_L, p_V are feedback coefficients; L and $\dot{L}$ are perturbed length and the rate of change of length of the tether, which meets the condition (4); $F_n(t)$ is the decelerating force.

This principle of regulating the length and the rate of its change (7) was used during the real orbital tether experiment "YES2" and in other research [18, 22, 28]. Here and below the modelling uses more simple principle based on kinematic control method, which is defined as

$$\dot{L}_p(\tau) = V_{\max} \cos^2(\omega\tau + \nu) \tag{8}$$

where V_{max}, ω, ν are parameters. Boundary conditions are imposed by the solution of the system of non-linear equations

$$\dot{L}_p(t_{fin}) = 0, \qquad \frac{d\dot{L}_p}{d\tau}(t_{\text{fin}}) = 0, \qquad \dot{L}_p(0) = V_r, \qquad \int_0^{t_{fin}} \dot{L}_p(t)dt = L_{\text{fin}}. \tag{9}$$

Parameters t_{fin}, $V_{\max}$, ω, ν are found by solving the system of equations (9). Due to the periodicity of equation (8), the system of equations (9) has more than one solution. Consider the solution where $\dot{L}_p(\tau)$ illustrates the method of deployment with areas of both acceleration and deceleration. The comparison of different methods of deployment has shown the effectiveness of the kinematic control method [29].

3.6. Numerical Calculation Results

The model of the tether system described above uses the model of elastic rod for the tether. This model provides with the basic knowledge of the motion of the aerodynamic tether system during its deployment. The following results are obtained for modelling a deployment of a tether system where the mass of the spacecraft is equal to 200 kg, the mass of the stabilizer is 12 kg, the initial altitude $H = 250$ km, Young's modulus for the tether material is $2.5 \cdot 10^{10}$ Pa, and the diameter of the tether is $0.6 \cdot 10^{-3}$ m. The characteristics of the tether refer to the Dyneema® fibre [30]. The initial velocity of the separation $V_r = 2$ m/s. For the tether system with final length of the tether equal to 15 km, the kinematic deployment method parameters are $V_{\max} = 3.5$ m/s, $\omega = 0.000325$, $\nu = 2.43$, $t_{\text{fin}} = 7040$ s.

The tension force oscillates during the first seconds of deployment due to shock loads caused by the control force. These oscillations are shown in figure 3. The value of tension force reached here (7.09 N) is the maximum tension during further deployment process for the systems with specified length of the tether and for the systems with a shorter tether as well.

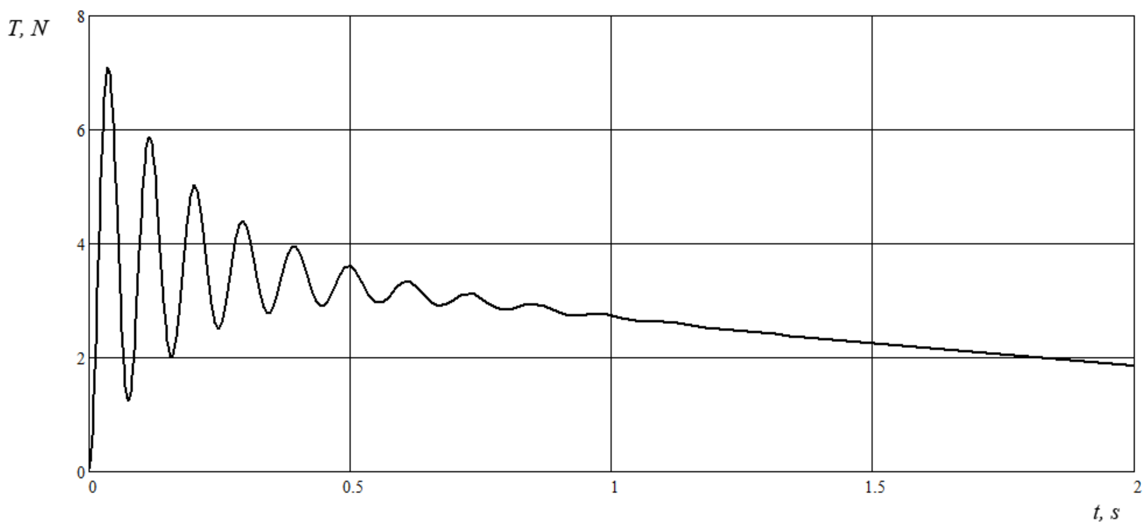

Figure 3. Tension force during the beginning of the deployment.

Figure 4 illustrates the trajectories of relative motion of spacecraft and the stabilizer during the deployment. The values of feedback coefficient are chosen under the condition of non-periodic process while the task of finding optimal values for the coefficients was not considered here.

Both bodies of the tether system may have static and dynamic asymmetry. The static asymmetry is the distance from the centre of mass of the body to the longitudinal axis of the body. The dynamic asymmetry is the difference between the moments of inertia I_{yi} and I_{zi}. The relative value of static asymmetry is defined using the diameters of the bodies and the relative value of dynamic asymmetry is the arithmetic mean of I_{yi} and I_{zi}. It was found that small static or dynamic asymmetry of the bodies does not have a significant influence on the motion of the system but there is a dependence of the angle of attack of the tether on the ballistic coefficient of the stabilizer. The higher the ballistic coefficient of the stabilizer, the lower the maximal values of angle of attack of the tether.

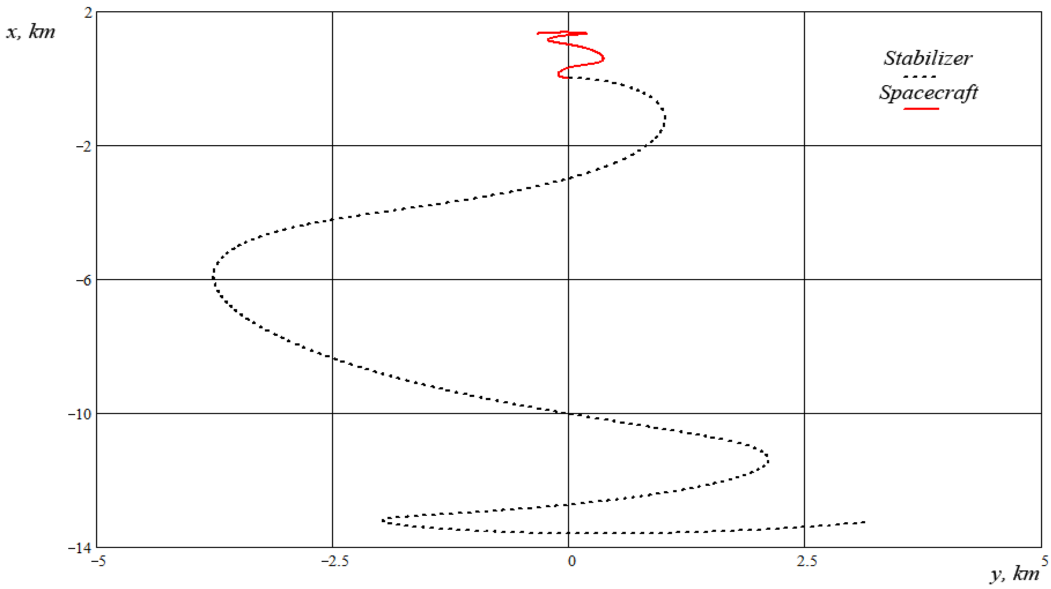

Figure 4. Trajectories of the bodies.

The model described above represents the tether as an inflexible rod – a one-dimensional stretchable mechanical connection without mass. This model does not take the mass of the tether into consideration, and since the tether is assumed to be straight, the elastic deformation and flexure of the tether itself cannot be taken into consideration. For more accurate modelling, it is necessary to use more complex multi-point model.

4. Modelling the tether as a multi-point system

4.1. Model Construction

To model the flexure of the tether it is necessary to split the tether into a number of parts. The higher the number of tether parts, the better the resolution of the model solution. Such a result would show the flexure of the tether system as a set of material points, or nodes, along the length of the tether. The anticipated non-straight line of nodes would indicate the tether response to constant or slow-time varying forces such as the gravity field and the aerodynamic forces. Therefore, the multi-point representation of the deployed tether system consists of n nodes, including two nodes for description of rigid bodies, the spacecraft and the stabilizer, and $n-2$ intermediate nodes defining the tether.

For modelling purposes, the full length of the tether is divided into parts according to the number of intermediate nodes on the tether. Starting from the beginning of deployment, as the tether unreels and the length of the tether reaches $l_{\text{full}}/(n-1)$, where l_{full} is the full length of the non-stretched tether, the new intermediate material point (node) is inserted into the mathematical model of the system (figure 5). The 10-node mathematical model of the tether system includes the following nodes: the spacecraft, 8 intermediate nodes and the stabilizer. The tether for this example is represented by 9 parts, and the lengths of the parts of the tether are equal under the condition that they are not stretched elastically. In other words, the tether is represented numerically, specifying a discretization of the tether into a series of material points (nodes) with elastic connections.

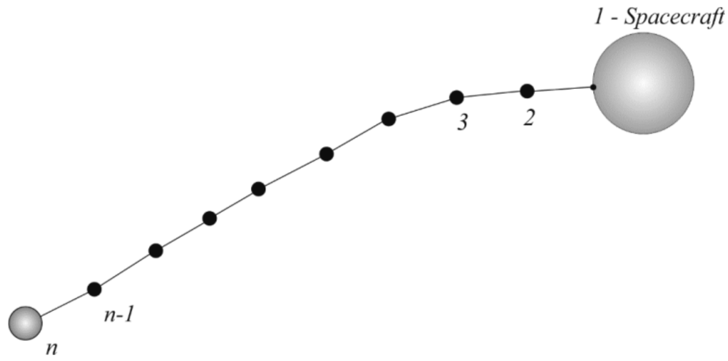

Figure 5. The multi-point model of the tether system.

For every intermediate node, the value of the tether tension is equal on both sides, and at the moment during the deployment that the node was created this value should have been equal to the tension of the tether in this point just before the node was inserted. The vector of velocity of the new node is defined based on the velocities of the neighbouring nodes using proportions. The velocity of the spacecraft after inserting the new node is corrected using of the law of conservation of momentum. It is assumed that the tether does not have internal dissipative forces like friction.

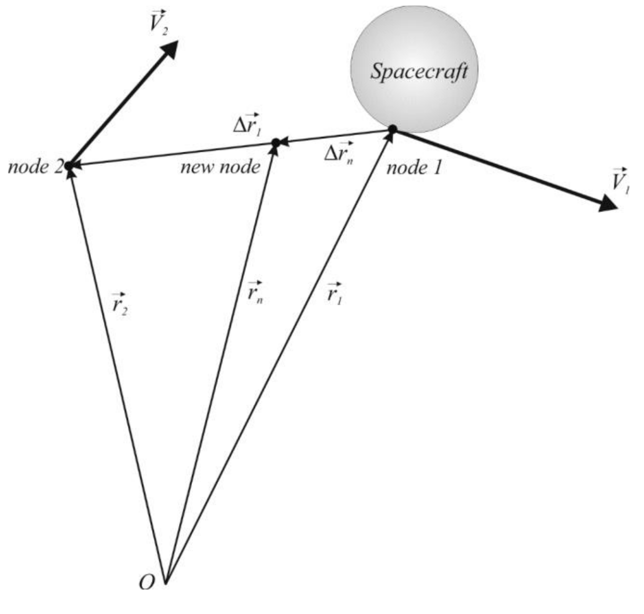

Figure 6. New node on the tether.

The multi-point model of tether describes a series of separations of material points from the spacecraft. These material points describe the stabilizer and the nodes on the tether. As the length of the non-stretched tether coming from the unreeling device reaches a pre-defined value equal to the distance between nodes on the tether, the next material point separates from the spacecraft. This new material point is added between spacecraft and previous node on the tether, or, if this were the first node on the tether, between spacecraft and stabilizer. The scheme of adding each new node is shown in figure 6. It is necessary to take into consideration that before the beginning of the deployment the tether is situated on the spacecraft, and the mass of the spacecraft decreases during the deployment as the tether unreels. The mass of the second body remains the same as in the beginning of the deployment process. When a new node is inserted into the mathematical model of the tether system, the mass of the spacecraft is reduced by the mass of the section of the tether.

4.2. Numerical Calculation Results

The advantage of the multi-point model is that it allows the elastic deformations and curvature of the tether to be calculated numerically. The following calculations are made for the tether system with the following parameters: mass of the spacecraft is equal to 200 kg, mass of the stabilizer is equal to 2 kg, radius of the spacecraft is 0.5 m, radius of the stabilizer is 2 m. drag coefficient for both bodies is equal to 2.4, Young's modulus for the tether material is equal to $2.5 \cdot 10^{10}$ Pa, diameter of the tether is $0.6 \cdot 10^{-3}$ m, the initial altitude is 250 km, and the full length of the tether is equal to 30 km. The deployment starts with initial velocity of separation $V_r = 6\ \mathrm{m/s}$. The parameters of the deployment process are defined by numerical solution of (9), and deployment ends at $t_k = 11555$ seconds. The multi-point modelling predicts angles having slightly smaller values than calculated using the two-point model because the aerodynamic forces acting on the tether are now included in the model. Therefore, the aerodynamic forces acting on the tether also have a stabilizing effect. Figure 7 depicts the change of the tension of the tether during deployment. The shock loads seen in the first 200 seconds are non-physical shocks and related only to numerical effects in the node adding algorithm. Ideally, the node adding algorithm should be modified to avoid this.

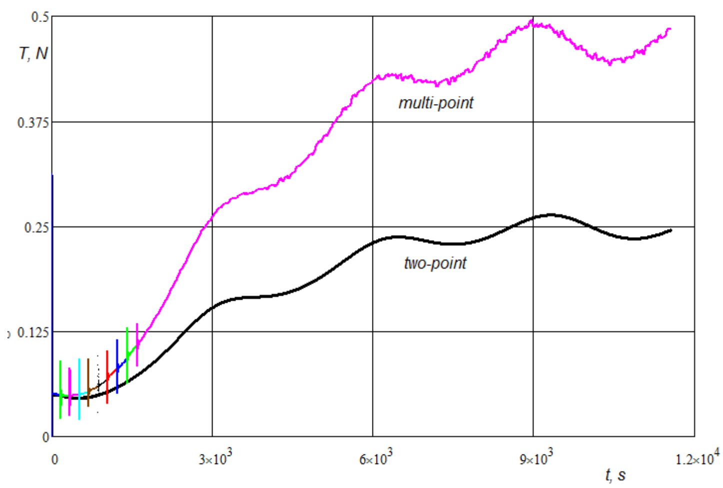

Figure 7. Tension of the tether, N.

The trajectory of the stabilizer relative to the spacecraft is shown in Figure 8. The solid line depicts the motion calculated using the two-point model, and the dashed line represents the motion predicted by the multi-point model. The spacecraft is placed in the upper right corner of the graph, and the stabilizer, affected by the aerodynamic and gravitational forces, moves away from the spacecraft as the tether is being deployed. The difference compared with the relative trajectory shown in figure 4 is based on the big difference in ballistic coefficients of the bodies in the second case.

The shape of the tether during the motion is of great interest. Figure 9 illustrates how the shape of the tether changes during deployment. Each circle on the tether depicts an intermediate node, and different lines show the state of the tether system at the moment of inserting each new node. The tether is almost straight at every moment of deployment, and only when the tether is almost fully deployed and is, therefore, long, is there is a slight curvature near the stabilizer end. A similar analysis was also made for a simpler model, with fewer intermediate points. By increasing the number of intermediate points from 5 to 8 it was found that, for this shape of the tether and specified deployment model, there is no significant difference in results of modelling parameters of the motion of the system. From this it

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can be concluded that further increase in the number of nodes is not advisable. Figure 9 also confirms that the altitude of the spacecraft decreases as a result of the influence of the atmosphere. The spacecraft is shown by the dot at the right-hand end of each line describing the tether, and its position goes down as the tether length is increased.

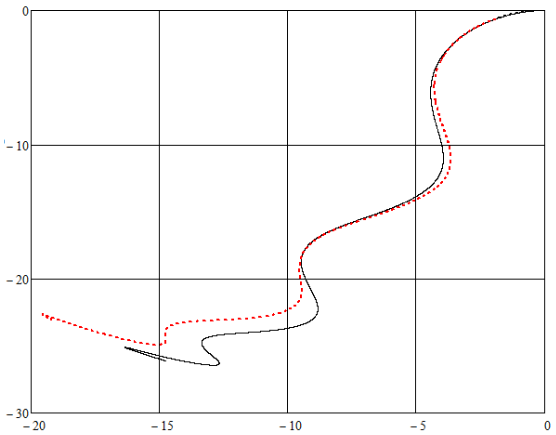

Figure 8. The trajectory of the stabilizer relative to spacecraft, length is indicated in kilometres.

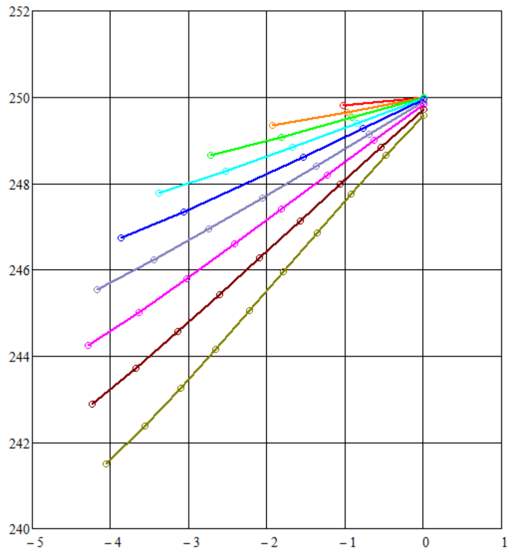

Figure 9. Shape of the tether, length is indicated in kilometres.

5. Peculiarities of multi-point modelling
A comparison of the results of modelling tether system deployment, for different tether lengths, using both the previous two-point model and the multi-point model was made. There is a difference in the motion parameters obtained by these two models. The tension of the tether calculated using the multi-point model is approximately twice that calculated using the two-point model, and the angles defining orientation of the tether system are also lower in this case. This is explained by the fact that the multi-point model represents the aerodynamic forces as acting not only on the stabilizer, but also on the tether, thus predicting a higher tension force than predicted by the two-point model, where only the stabilizer aerodynamic forces were modelled. According to the properties of material from which the tether is made, the value of tension force is admissible in both cases.

The multi-point model is more computationally expensive than the two-point model. As the results obtained by the more complex method agree with those obtained by two-point model, it can be concluded that the two-point model can be relied on, not only for the particular modelling of the deployment of the tether system, but also for automated calculations and selection of parameters of the tether system at the design stage. On the other hand, the multi-point model does show the motion of the tether and the tether system as the system with distributed parameters. When the shape of the tether is close to the straight line, it is possible to reduce the number of intermediate nodes used.

During the real process of the deployment, there can be emergency situations such as a breakage of the tether or jamming in the unreeling device. In these cases, the multi-point model would allow the parameters for the resulting motion of the system to be calculated: the two-point model would not be capable of providing this information about the flight.

6. Discussion
The descent of a tether system from orbit includes two stages. During the first stage the tether system is deployed, and the second stage describes the motion of the deployed system in the atmosphere. There are similar problems in the modelling of the tether motion in both of these stages. The first group of

problems refer to the manufacture process, the second group to the perturbations arising from the effects of the surrounding medium, and the third to perturbations not taken into consideration in the mathematical model of the motion. In the manufacture of any product it must be assumed that there can be design deviations arising from inaccuracy, lack of quality control or comparatively high manufacturing tolerances. Examples of such deviations include mass and shape asymmetry of the spacecraft, flaws in the unreeling mechanism or non-uniformity of the tether material. These deviations lead to the perturbations in parameters of the tether system. Simplifications made in the modelling of the atmosphere account for the surrounding medium effects. Existing models of the atmosphere do not include the chaotic character of local flows. The most vivid example of atmosphere perturbations is wind. Wind is a natural phenomenon and takes place in every location on Earth, but it is highly problematic to take it into consideration. There are also differences in the properties of the atmosphere resulting from, for example, time of day, season, and solar activity.

These influences can lead to perturbations during the deployment process or in the motion of the deployed system. For example, there can be errors in the absolute value or direction of the velocity of separation at the beginning of the deployment, and so the intention is that the model of deployment overcome such perturbations, by making use of stabilization methods. Methods such as spinning, or the use of a tether system provide the means of achieving the conditions of stable motion.

The magnetic field of the Earth was not taken into consideration in this research but depending on the materials the spacecraft is made from there can be an influence from appropriate forces. There are other forces acting on the spacecraft during its motion in the atmosphere, including the pressure from the sunlight, gravity forces from space objects and so on. The magnitudes of these forces are small and are usually not included in the mathematical models. It is also necessary to consider the mutual influence of the spacecraft and stabilizer bodies on each other. The perturbations in airflow caused by the presence of one body changes the regime of airflow on the body downstream, thus changing the values and directions of the aerodynamic forces.

Every real system has dissipative forces acting on or within the system, and damping is of great importance during the motion. Damping reduces the amplitudes of oscillations thus tending to make motion more stable. The dissipative forces cannot destroy the stability of motion of the system, but it is important to include damping to get the mathematical model closer to the real system. The problem here is the damping is one of the most complex processes for mathematical modelling, and the appropriate choice of damping model for use in the modelling of space tether systems is the subject for ongoing research [8, 31].

Another important consideration in the tether system design is the cross-sectional shape of the tether. The Dyneema® fibre considered here has a circular cross-section; however, the choice of a tape cross-section would make the tether less vulnerable to micrometeorites attack [8]. Other advantages of a non-circular cross-section could include aerodynamic characteristics that are dependent on the angle between the tether surface and oncoming airflow. In such circumstances it would be necessary to include the localized rotation of the tether with respect to its longitudinal axis.

7. Conclusions

The multi-point model of the tether system allows the examination of the deformation and flexure of the tether during and after the deployment. Compared to the two-point model, the multi-point model takes into consideration the mass of the tether and the aerodynamic forces acting on the tether. For modelling purposes, these forces were treated above as acting on the nodes.

The tether can be used as an aerodynamic stabilizer itself, and the small spacecraft can be stabilized by using a tether represented for modelling purposes by number of nodes during its motion in the atmosphere. Compared to the two-point model, the multi-point model predicts more accurate values for the parameters of the tether system during the motion through the atmosphere. The number of intermediate nodes during the modelling was set to eight: increasing the number of nodes leads to significantly higher model complexity without giving any noticeable improvement in the result accuracy compared to the model with fewer intermediate nodes.

The comparison of results obtained from the two-point and multi-point models show that the change in mass after adding each node and the influence of aerodynamic forces acting on the tether leads to differences in values of the tension force and angles describing the orientation of the bodies. Therefore, the multi-point model has noticeably higher accuracy compared to the models of the rigid and elastic rods. In addition to modelling the motion of the deployed system, the multi-point model also enables the modelling of the deployment process taking into consideration the curvature of the tether.

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