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Abstract. Micro-electro-mechanical systems (MEMS) devices are widely used in industry. The micro-actuator is an important component of such devices, transforming external influence into mechanical displacement. The development of a design technique to derive the optimal design parameters for a micro-actuator is a problem of current interest. The present paper describes a technique for determining geometric parameters for a simplified micro-actuator structure (a hemispherical shell), such that under a prescribed critical pressure it will undergo a specified discrete deflection. Such a deflection is commonly referred to as "snap-through". A mathematical model and a Finite Element procedure for the mechanical analysis of a flexible thin-walled shell under large deformation are proposed. Initially the snap-through is modelled as a quasi-static effect, but subsequently, the influence of the inertia is also considered. The optimization procedure was performed using the PSE/MACROS optimization program. The results for an example model numerical optimization are shown.

1. Introduction

The development of innovative base elements for micro-electro-mechanical systems (MEMS) devices has significant potential for the technological modernization of industry. The development of new MEMS device designs is predicated on the development of methodologies for analysis and synthesis of constitutive elements, each with given specific performance characteristics. Each MEMS device comprises a sensor sub-system, a sub-system for analysis and decision making, and an actuator sub-system [1]. This paper focusses on the mechanical aspects of the actuator sub-system.

For a MEMS device to operate efficiently and effectively, the mechanical performance of the actuator sub-system has to be finely tuned. For efficiency, the actuator must be low inertia, and be capable of large motion for low input energy. It must also be capable of moving from one state to another, with minimal lag, and avoid lingering in an intermediate state. For this reason, elastic snap-through structures are ideal: such structures have two (or more) stable states, and only a small applied force is needed to trigger a change of state. A contact lens for vision correction is an example of an everyday object with this kind of performance. Mechanical analysis of such structures is not without its complications, because a small variation of applied load does not lead to a simple small variation of deformation: the structural performance is not linear. This type of non-linear behaviour is known as "non-linear deformation".

There is a well-established theoretical foundation in the field of non-linear deformation of thin-walled structures. The non-linear behaviour of thin-walled shells and snap-through has been documented by many authors [2-8]. More recent research papers [9-10] describe computational

algorithms for non-linear deformation. In combination, this development of technology has provided new avenues for research, including the optimization of actuator sub-systems for discrete action MEMS devices. The utilisation of a flexible thin-walled shell element, made of novel materials [11, 12] (*e.g.* silicon), as an actuator is thought to be innovative.

In the present paper, a technique is presented, for determining geometry parameters for a discrete action micro-actuator, operating under snap-through. To keep things simple, the geometry is a hemispherical shell of constant thickness. Note that the radius of the support circle remains constant as it is specified by the device. The parameters in this case are hemispherical radius and shell thickness, subject to particular pressure application conditions and a requirement to achieve a predetermined deflection. In the first instance, the study assumes the snap-through to be a quasi-static effect, but subsequently, the influence of the inertia is also considered. A mathematical model, based on the Reissner thin-walled elastic shell theory [7], for the analysis of flexible shell structures under large deformation was implemented in computer code written by the author, and validated in ANSYS finite element software. A parameter subspace changing method, developed by Gavriushin [13], based prior work on computational algorithms for shells by Valishvili [14], was used to solve the non-linear problem [15]. The optimization was performed using the PSE/MACROS optimization program.

2. The method adopted

The hemi-spherical thin-walled micro-actuator is designed to perform mechanical switching. Let us suppose that its deflection is required to take a given value d^* when the critical pressure P^* is applied. The elastic characteristic of the micro-actuator which includes a snap-through mechanism and the deformed shape of the actuator are shown schematically in Figure 1.

The shell straightens after applying pressure. This part of the process is illustrated by the DA part of the curve. The shell instantly changes the direction of the deflection at the point A (the upper critical point) – the snap-through at point A. Then the shell continues its deformation from the point B of the stable part of elastic characteristic. The deflection of the shell decreases if the pressure decreases (BC) until the point C (lower critical point) and the CD snap-through.
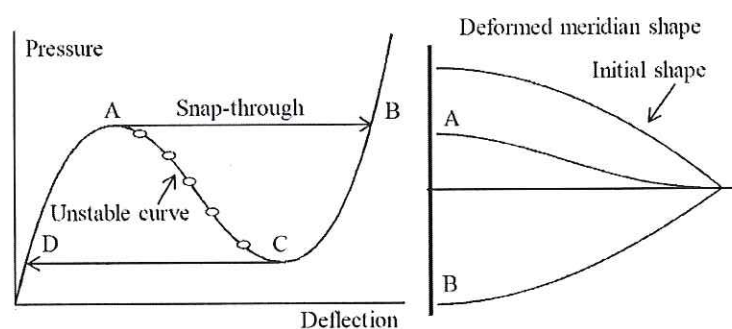

Figure 1. Schematic of Micro-actuator discrete elastic response and deformed shape.

The initial data for the research is given in Table 1. The required outcome of the optimisation process is the selection of values R, hemi-sphere radius of curvature, and h, thickness, subject to the values of critical pressure P^* and the corresponding deflection d^* being equal to customer requirements. The acceptable ranges of variation of these parameters are given in the Table 1. The functional relationships between the criteria and parameters are not set explicitly, but are calculated using Finite Element [16, 17, 18], in ANSYS as described below.

The problem is solved using the axisymmetric element formulation. The shell is modelled using PLANE182 elements. These elements have 4 nodes with 2 degrees of freedom at each node. The

material of the shell is linearly elastic. The shell is simply-supported around the edge and is uniformly loaded by pressure. The analysis is performed using the parameter continuation algorithm arc-length method for a problem with large displacements [19, 20]. In this method the length of the arc along the elastic characteristic curve is used as a parameter in the parameter continuation method, rather than displacement (or pressure) in displacement- (or force-controlled modes). This makes it possible to overcome calculation difficulties. The PSE/MACROS program is used for the solution of the optimization problem [21].

In the flowchart detailing the algorithm (Figure 2), the Optimizer block contains information about the parameters, constraints and objective functions. This block governs the solution process.

There are two objective functions, one in terms of applied pressure and the other in terms of snap-through displacement. These were specified as

$$F_1 = |P - P^*| \qquad\qquad F_2 = |d - d^*|. \qquad\qquad (1)$$

Table 1. Problem specification data.

Parameter		Value
Support ring radius	r_s	0.25×10^{-3} m
Hemi-sphere radius	R	$2.8 - 3.2 \times 10^{-3}$ m
Shell thickness	h	$2 - 5 \times 10^{-6}$ m
Desired pressure	P^*	50×10^{-3} Pa
Desired displacement	d^*	24×10^{-6} m
Young's modulus	E	150×10^9 Pa
Poisson's ratio	ν	0.23

Table 2. Viable geometry parameters.

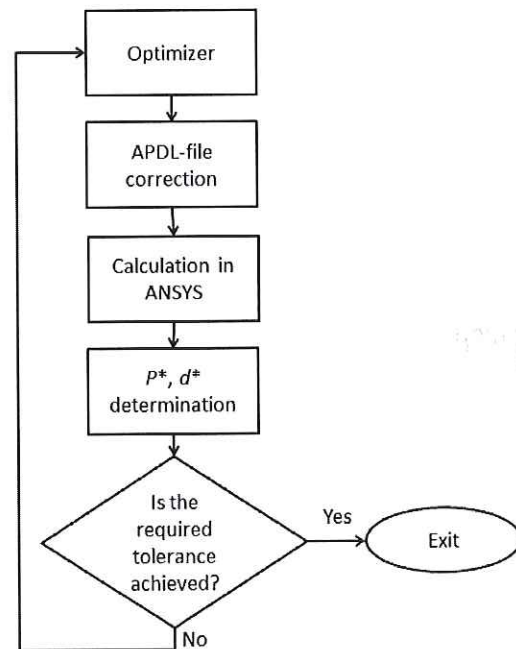| Number	$R \times 10^{-3}$ m	$h \times 10^{-6}$ m
1	3.140	2.234
2	3.127	2.242
3	3.119	2.244
4	3.098	2.229
5	3.097	2.228
6	3.093	2.223

Figure 2. Flow chart of the solution in PSE/MACROS program.

An initial file is written in ANSYS Parametric Design Language (APDL), which performs the calculation within the ANSYS program. This file is held open and updated during the process of optimization. At each iteration, new values for the variables R (hemisphere radius) and h (shell thickness) are substituted into the APDL file, and the ANSYS analysis is repeated. The calculated values for the critical pressure, P, and the corresponding deflection, d, after the snap-through are extracted from the ANSYS results database using a subroutine written using Python language [22]. The Optimizer continues to vary R and h until the values of P and d match those of P^* and d^* to within the tolerance requirements.

3. Results obtained using a quasi-static modelling approach

In this optimization problem, 80 combinations of the two geometric parameters were considered during the solution of optimization problem.

A candidate solution is considered viable if it is near optimal in both objectives. Post-optimization, a further constraint might also be applied, $d \geq d^*$, to ensure that in the actuator the deflection is sufficient to ensure that a contact is closed. The 80 candidate solutions are projected onto the $P - d$ plane (Figure 3), allowing the user to choose the anticipated solution [23, 24, 25]. In this figure, there are six viable solutions, which are displayed in the in-set box [26]. The values of the variables R (hemisphere radius) and h (shell thickness) are displayed in Table 2, but it can be clearly seen that these are essentially identical, and the optimization solution is converged. The resulting shape, before and after snap-through is shown in Figure 4.
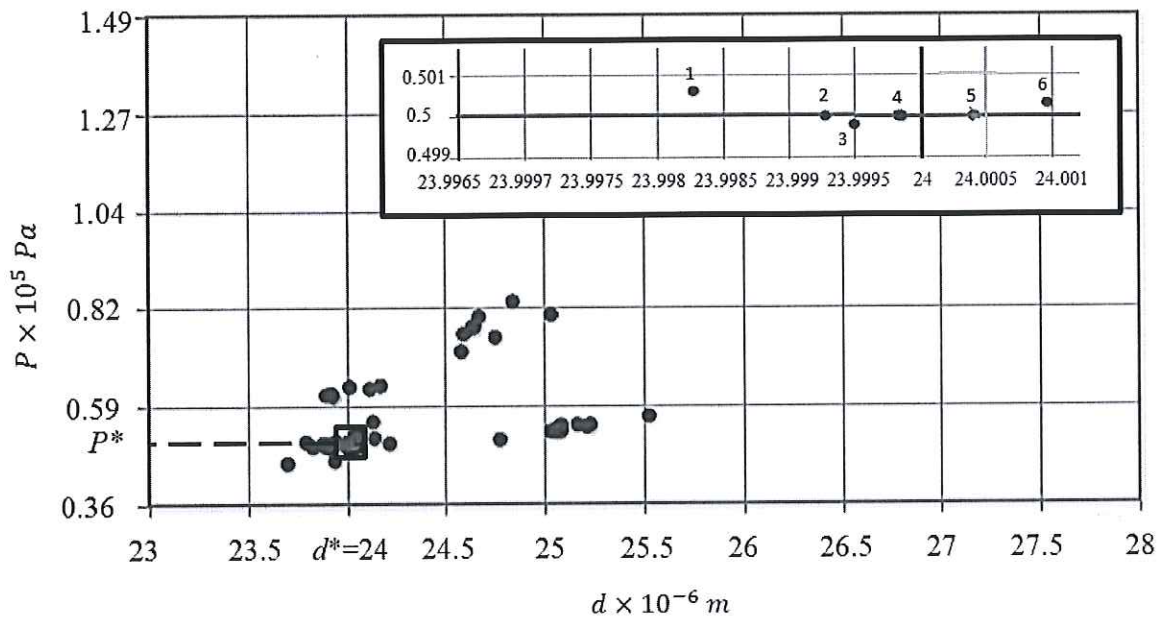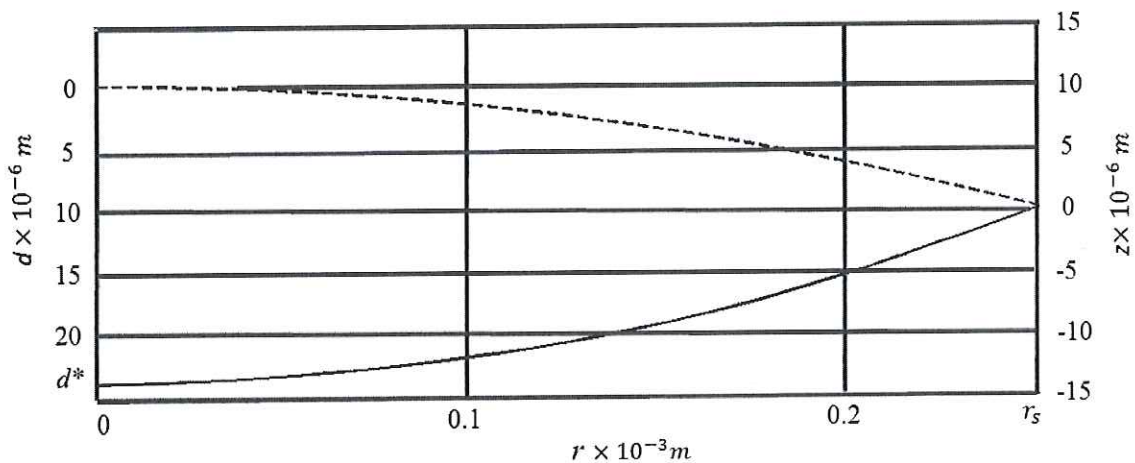

Figure 3. The $P - d$ plane.

Figure 4. Initial shape (dashed) and shape after snap-through (solid) for optimised geometry.

4. Results where inertial effects were also considered

To determine whether it would be necessary to improve the Finite Element model of the actuator, the effect of inertia during the snap-through event was also considered. In this case it is again necessary to find viable geometrical parameters, and then to examine whether these differed significantly from those obtained assuming quasi-static deformation.

The procedure of solution for inertial loading in ANSYS is broadly similar, with the significant difference that in this case, the problem is solved incrementally in time, so it requires significantly greater computing power. This is because the explicit FEA formulation was used, which is "conditionally stable", hence the size of the time step increment is fixed by minimum element size. Because the FEA is time-consuming, the surrogate based optimization (SBO) method in PSE/MACROS program was used. Surrogate methods substitute an approximation for the computational model prediction, interpolated from results computed at a number of points in the problem domain. This is appropriate where the surrogate model gives a reasonable approximation of the model behaviour, and reduces the computational demand during the optimisation cycle [27].

For the solution of the optimization problem 17 combinations of geometric parameters were required, yielding optimal geometry values of $R = 2.94 \times 10^{-3}$ m and $h = 2.24 \times 10^{-6}$ m. The value for thickness is close to that obtained using the quasi-static modelling approach (less than 0.5% variance). The value for the hemispherical radius differs by approximately 5%, indicating that although inertia does influence the performance, it is quite a small effect, therefore it might not be necessary to take inertia into account during the initial phase of optimization. This could make the optimization process more efficient.

5. Discussion

The current research is at an early stage in development, and the focus here has been to develop an effective methodology for parameter optimization for structural components that display non-linear elastic large deflections. For this reason, the multi-objective optimization has been restricted to just two objectives, and the geometrical variables to just two parameters. Despite this simplification, the optimization problem has shown sufficient richness to demand further investigation.

PSE/Macros is a commercial off-the-shelf engineering design optimization package. As such, it is designed to be used by non-experts, to perform robustly, and to provide user-accessible results. It is neither under obligation to provide the user with details as to how it obtained those results, nor to enable the user to drive its operation in any particular way. The "black box" nature of the package restricts the user, such that it is difficult or impossible to deduce which optimization algorithms are being deployed, and impossible to impose a particular methodology or utilise a user-written algorithm.

For research purposes, it is necessary both to understand the processes of optimization, and to explore and prescribe particular actions and algorithms, firstly in order that the work might be validated by other researchers, and secondly to develop and improve the optimization algorithms for this particular class of optimization problem. For this to be feasible, the present authors recognize that future optimization work should be performed using either an open-source optimization package, or by developing in-house capability.

The refinement of the modelling capability to take into account the inertia effects of the actuator made some significant demands on the Finite Element computational time. To overcome this, the optimization step included surrogate modelling, and in doing so, the number of optimization steps required dropped from 80 to 17. Given the stochastic nature of optimization algorithms, it would be unreasonable to draw a firm conclusion from this single example, but this result would suggest that surrogate modelling could prove to be extremely valuable as the level of model geometry complexity is increased.

Optimization is currently a very busy research field, and there are very many new algorithms and techniques under development, for example [18, 23, 24, 25, 27, 28 29]. It is already known to concatenate algorithms; first to explore the parameter domain in order to determine how strongly

variation of parameters influences the objective functions and to identify a region of the domain where a strong optimum is likely to be found, and subsequently to drive efficiently towards that optimum.

There are two particular areas of optimization research that would have particular relevance to the further development of micro-actuator design methods: the use of Pareto front mapping and penalty functions [24]. While a robust optimization technique might generate a number of viable design options, it would remain the duty of a human designer to make the final geometry selection. The present study has been a simplified paradigm of a much more complex problem: the choice between six options can be made on a case by case basis. Where the numbers of objectives is much greater, for example to include cost models for manufacture, raw material costs, materials utilisation, fatigue performance, *etc.*, Pareto front mapping could prove a useful tool in trading between the characteristics of different types of optima. In the measurement of the degree of an optimum there can sometimes be asymmetry. For example, in the present study the objective function for deflection was defined by minimising the difference between the deflection achieved by the candidate geometry, and the desired deflection. In point of fact, a slight over-deflection would probably be acceptable, while an under-deflection would mean that a contact would not be closed, and this would be entirely unacceptable. Penalty function methods can be applied to such asymmetric objectives, thereby screening out the candidates that do not meet practical requirements.

Analysis of the tolerance domain in the region of the "ideal" geometry parameters is also of significant importance. For a MEMS device, it is important that it operate without error, which means that every device must operate in the same way despite the inevitable geometric variability that would exist between individual devices. Should a high degree of tolerance be demanded, the cost of manufacturing would be inhibitive. Design optimization should therefore seek not for the absolute optimum, but for a robust optimum.

6. Conclusion

A technique for determining geometric parameters for a discrete action micro-actuator, to translate a prescribed critical pressure into a predetermined value of deflection has been described. The actuator was simplified to be a hemispherical shell, with a simply supported ring support of fixed diameter. The two geometry variables considered were the hemispherical radius, and the shell thickness. The mechanical analysis was performed using Finite Element Analysis. The mechanical model adopted was initially assumed to be quasi-static, but subsequently refined to consider inertia effects. The optimization process was performed using an off-the-shelf engineering design optimization package. The proposed methodology followed has been demonstrated to be effective, a number of opportunities for methods development have been identified, and these will form the basis for further more challenging design optimization studies.

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